THE BIRTHDAY PROBLEM

The Birthday Problem is a very famous probability problem that illustrates the use of The Multiplication Rule and The Complement. It (indirectly) states that all you need is 23 people in order for the probability that at least 2 share a birthday to be greater than 0.5. Or, P(at least 2 share a birthday) > 0.5.

Here is why: (disregarding leap years)

\[ P(\text{person 1 has a birthday}) = \frac{365}{365} \]

\[ P(\text{person 2 has a different birthday}) = \frac{364}{365} \]

\[ P(\text{person 3 has yet a different birthday}) = \frac{363}{365} \]

\[ P(\text{person 4 has another different birthday}) = \frac{362}{365} \]

\[ \ldots \]

\[ P(\text{person n has still a different birthday}) = \frac{365 - n + 1}{365} \]

Now, by the Multiplication Rule,

\[ P(\text{all n birthdays are different}) = \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \ldots \frac{365 - n + 1}{365} \]

And by The Complement,

\[ P(\text{at least one shared birthday}) = 1 - \left[ \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \ldots \frac{365 - n + 1}{365} \right] \]

\[ = 1 - \left[ \frac{365!}{(365 - n)!} \cdot \frac{1}{365^n} \right] \]