Definitions and Properties of Exponents

The following assumes that no denominators are 0 and that $0^0$ is undefined.

1 as an exponent: $a^1 = a$

0 as an exponent: $a^0 = 1$

Negative exponents:

$$a^{-n} = \frac{1}{a^n}$$

$$\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

The Product Rule: $a^m \cdot a^n = a^{m+n}$

The Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$

The Power Rule: $\left(a^n\right)^m = a^{mn}$

Raising a product to a power: $(ab)^n = a^n b^n$

Raising a quotient to a power: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

If $m$ and $n$ are integers, then the bases can be arbitrary real numbers.
If $m$ and $n$ are rational numbers, then we need to avoid even roots of negative numbers.
If $m$ and $n$ are real numbers, then we restrict $a$ and $b$ to be positive numbers.