Characteristics of a Neutron Star

— Mass about 2 solar masses
— radius about 10 km
— like a huge nucleus
— density is the same as the density of an atomic nucleus
— because they start big with a slow rotation, the end up small with a rapid rotation — about 30 times a second.
— material of the neutron star behaves like a superconductor
— magnetic field of the neutron star is trapped in the superconducting matter and rotates around with the star

We have found neutron stars — first were pulsars that sent rapid pulses of the light in our direction.

The magnetic field axis in a neutron star will probably be off the spin axis — case here on Earth.

Charged particles trapped in the magnetic field send beams of light along the axis of the magnetic field.

As the neutron star spins, the beam of light sweeps around. When pointed at Earth, it sends a pulse our way. We see a series of pulses, say, 30 times a second.

This is the lighthouse model of a neutron star/pulsar.

We can also detect neutron stars in close binary systems — neutron star is close enough to its companion to bleed mass off of it.

As the matter loses gravitational energy, it converts it to x rays — we can detect the x rays. We know it’s a neutron star because we know what sort of x ray spectrum we should see from a neutron star.

If it is a not close binary system, and we calculate the mass of the unseen companion and if it has the mass of a neutron star, we know it’s a neutron star.

Gravitational Lens Effect — If a neutron star comes between us and distant object, it will cause the distant object to increase in brightness. How much and how long tells us the nature of the impeding object and, with the right properties, it will be a neutron star.

What if the final mass of the remnant is greater than 2 to 3 solar masses (main sequence mass of more than 40
solar masses)?

Not even the Pauli repulsion of the neutrons can prevent further contraction. Gravity takes over and the object contracts without stopping.

What goes on until the object gets to be atomic size or so is governed by the general theory of relativity — GR.

Special Theory of Relativity — SR

Michaelson-Morely Experiment

Hypothesis: electromagnetic waves travel through a medium called the ether.

M-M set about finding the motion of the Earth relative to the ether.

Compared the speed of a light signal along perpendicular paths. Light from the source $S$ travels toward mirror $M_1$, which is a half-silvered mirror. Some of the light passes through toward mirror $M_3$, where it is reflected back. Some of the light is reflected toward mirror $M_2$, where it is reflected back. The two beams of light are recombined at mirror $M_1$, and observed by the observer. If the speed of light is different along the two paths, the observer will see the effect of interference between the two beams. If the ether flows, say, horizontally the right, the horizontal light beam will take less time to complete the trip than the vertical one will. One should see interference and from the amount of interference, the speed of the ether can be deduced. No interference was seen.

Found no difference in the speed of light in the two different directions.

Two Postulates for the Special Theory of Relativity:

1. The laws of physics are the same in all unaccelerated frames of reference.

2. All observers, regardless of their frame of reference, will measure the same value for the speed of light.

Consequences.

1. Simultaneity

Two events that are simultaneous in one frame of reference are not necessarily simultaneous in any other frame of reference.

Note that, if two events occur simultaneously at the same point in space, they will be
simultaneous in all frames.

2. Time Dilation

Moving clocks run slow.

\[ \Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} , \]

where \( \Delta t \) is the time between ticks in the moving clock, \( \Delta t_0 \) is the time between ticks for the clock at rest, \( v \) is the speed of the clock, and \( c = \text{speed of light} \).

Has been tested:

1. Atomic clocks, synchronized, and then one sent on a plane trip for 24 hours, brought back disagree just as shown above.

2. GPS takes this into account.

3. Muons are unstable particles with a lifetime of 2.2 \( \mu s \). They last longer when their lifetimes are measured while they are in motion. The time dilation relation predicts, within experimental error, what is observed for the lifetime of the moving muons.

Length Contraction

Muons are created in the upper atmosphere due to collisions between cosmic rays and molecules of air. These muons get to the surface of the Earth and form about 75\% of the background radiation at the surface.

But traveling a the speed of light for 2.2 \( \mu s \), they would only be able to travel a few hundred meters and wouldn’t make it to the surface of the Earth. They get here due to time dilation—the rapidly moving muons last long enough to make to the surface.

Consider the rest frame of the muons — in this frame, they last for only their 2.2-\( \mu s \) lifetimes with the Earth rushing up to meet them at near the speed of light.

The Earth is only going to move a few hundred meters in the 2.2 \( \mu s \) — how does it reach the muons before they decay?

The distance from the Earth to the muons, in the muon rest frame, has shrunk or contracted to a few hundred meters.

Moving rods are shorter.

\[ L = L_0 \sqrt{1 - \frac{v^2}{c^2}} , \]
where \( L_0 \) is the rest length of the rod and \( L \), the moving length.

**General Theory of Relativity — GR**

Two Postulates of the General Theory of Relativity:

1. The laws of physics are the same in all frames of reference.
2. The principle of equivalence.

Three statements of the principle of equivalence:

1. The inertial mass (the one that appears in Newton’s second law, \( F = ma \)) is the same as the gravitational mass (the one that appears in Newton’s law of gravity).
2. No experiment can be done in a closed elevator that will distinguish between being in free fall in a gravitational field or moving with constant velocity in gravity-free space.
3. No experiment can be done in a closed elevator that will distinguish between being at rest in a gravitational field or having a uniform acceleration in gravity-free space.

Consequences of the principle of equivalence (#3)

1. Gravity is a manifestation of the curvature of space.

Follow the path of a light pulse in an elevator accelerating in gravity-free space. The dashed line in the figure at right shows the path of the pulse as observed by an observer who remains at rest outside the elevator. Each different color rectangle represents the position of the elevator at the given time. The \( \times \)'s represent the locations of the pulses at each given time.

The next figure shows the path of the pulse as seen by an observer inside the elevator. This observer sees that the pulse falls from the upper left corner of the elevator to the lower right corner.

By the principle of equivalence, the same thing must happen in an elevator at rest here on Earth. (Note that we don’t see this effect because of the large value of the speed of light. It travels so fast that it doesn’t fall a measurable distance in the Earth’s weak gravity.)

We see that gravity has an effect on light—bends the path of the light.

The interpretation is that gravity bends spacetime and that light follows the curvature of space.
Einstein showed that it is the presence of matter and energy in space that causes spacetime to curve.

Einstein’s equations are equations for the metric of spacetime.

Einstein’s Field Equations — schematically:

Curvature = mass and energy

Consider the Pythagorean theorem:

\[ a^2 + b^2 = c^2 \]

Not true in this form on a curved surface. On a curved surface, we have this:

\[ g_{aa}a^2 + g_{ab}ab + g_{ba}ba + g_{bb}b^2 = c^2 \]

The \( g \)'s are called the metric of the space. The two middle ones are always the same and one can always choose a frame in which they are zero.

Einstein’s field equations are equations for the metric—once we have the metric, we can predict the paths of particles and light through the space.

 Analogies:

1. Imagine a surgical rubber sheet stretched over a wooden frame. Place a large ball bearing on the sheet, and it will stretch the sheet. A second small ball bearing placed on the sheet next to the first ball bearing will roll toward it as if there were a force—gravity!—between the two.

2. Consider two bodies traveling with constant speed along two lines of longitude, starting at the equator and heading toward the north pole. As they travel, we see them move closer together because the two lines of longitude intersect at the north pole. From our vantage point in three dimensions, we easily see this to be the case. But a two-dimensional observer on the surface of the sphere won’t be aware of the third dimension and will interpret this moving together of the two bodies as a force—gravity!—between the two.

GR is based on the idea that gravitational fields and
accelerations are equivalent — Principle of Equivalence.

If we do an experiment in an accelerating spaceship we will get same results as if we did the experiment sitting at rest in a gravitational field — say, at the Earth’s surface.

Another result of GR — time runs slow in a gravitational field.

Put John at the bottom of the spaceship and Marsha at the top.

John and Marsha both have clocks.

At time zero on both clocks, two things happen — the spaceship begins to accelerate and John sends a pulse of light toward Marsha.

The light pulse moves toward Marsha and eventually reaches her. She sees this as the first tick on John’s clock.

A time 1 second on John’s clock, he sends a second pulse. Since Marsha is now moving away at some speed, it will take longer for this second pulse to reach Marsha than it did for the first pulse to reach Marsha.

Marsha measures the time between pulses to be greater than 1 second, that is, greater than the time that John measures.

Marsha concludes that John’s clock is ticking more slowly than once a second. His clock runs slow.

The same thing must happen for an experiment done at rest on Earth.

GR has been tested in a number of different ways and has always met predictions.

1. Bending of starlight near the edge of the Sun during and eclips is correctly predicted by GR.

2. Due to time dilation, GR predicts that laser light should drop in frequency as it travels from the basement to the attic of a building. The experiment agreed with the predictions of GR.

3. GR correctly accounts for a 43 seconds of arc per century precession of Mercury’s orbit.

4. GR correctly accounts for a degrees per year precession in a binary neutron star system.

5. There are some competing theories to GR—so called scalar/tensor theories. Predictions about the characteristics of binary systems in which on object is much more massive that the other are different in GR and these theories. Recently, a binary system containing a neutron star and a white dwarf where the ratio of the masses is about 12 were measured, and the results predicted by GR agreed with these observational results.